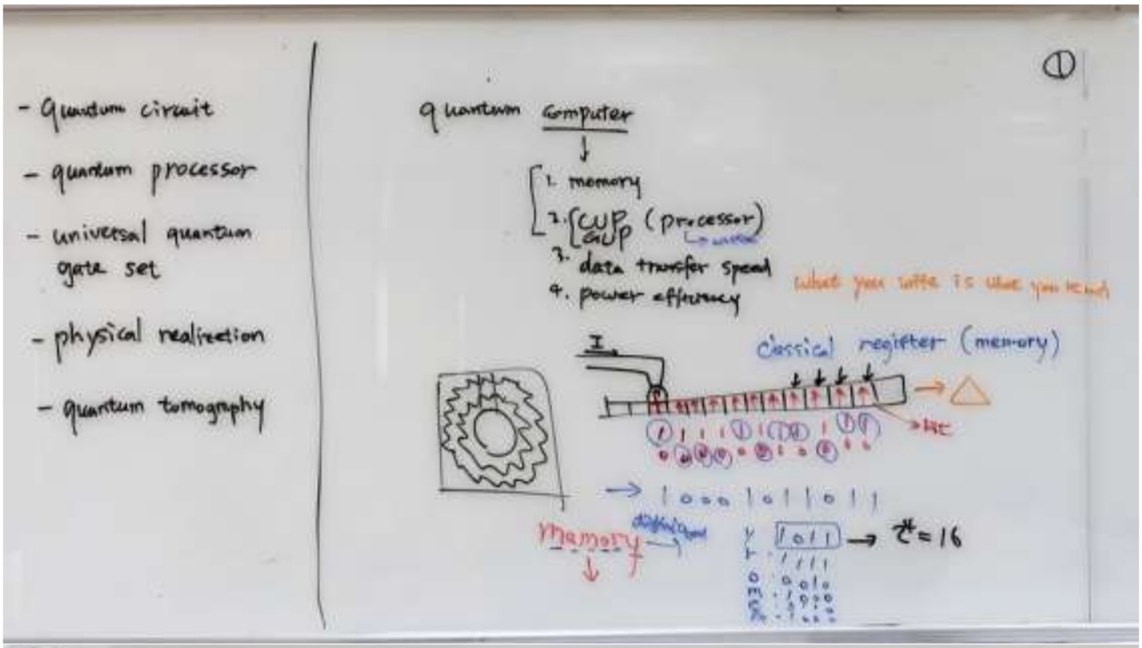
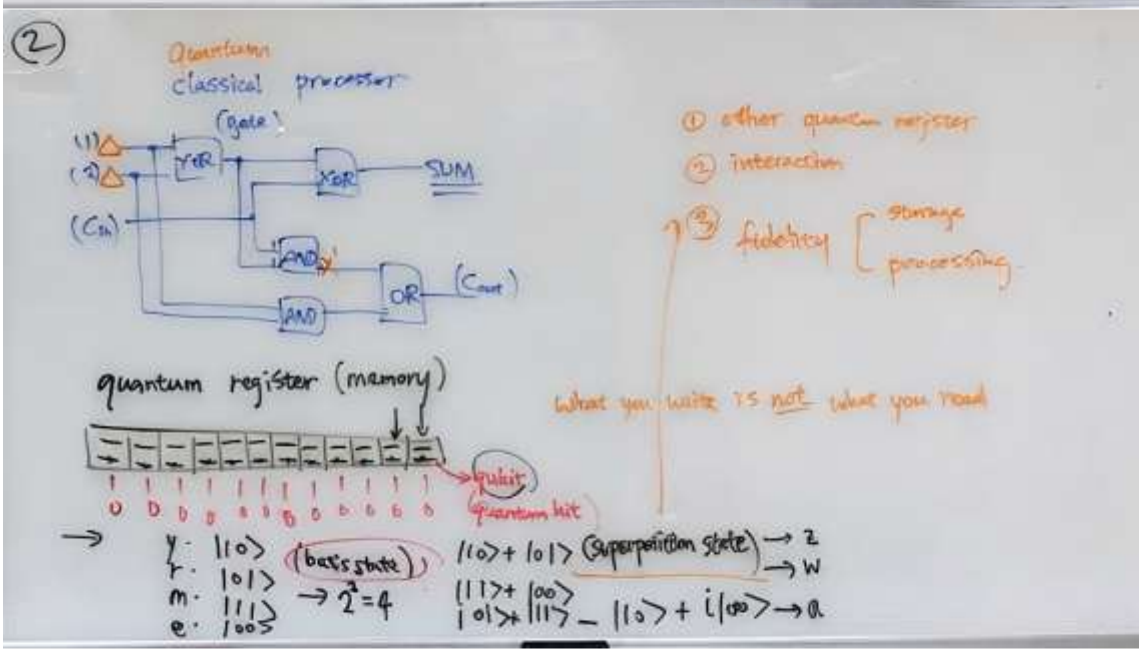
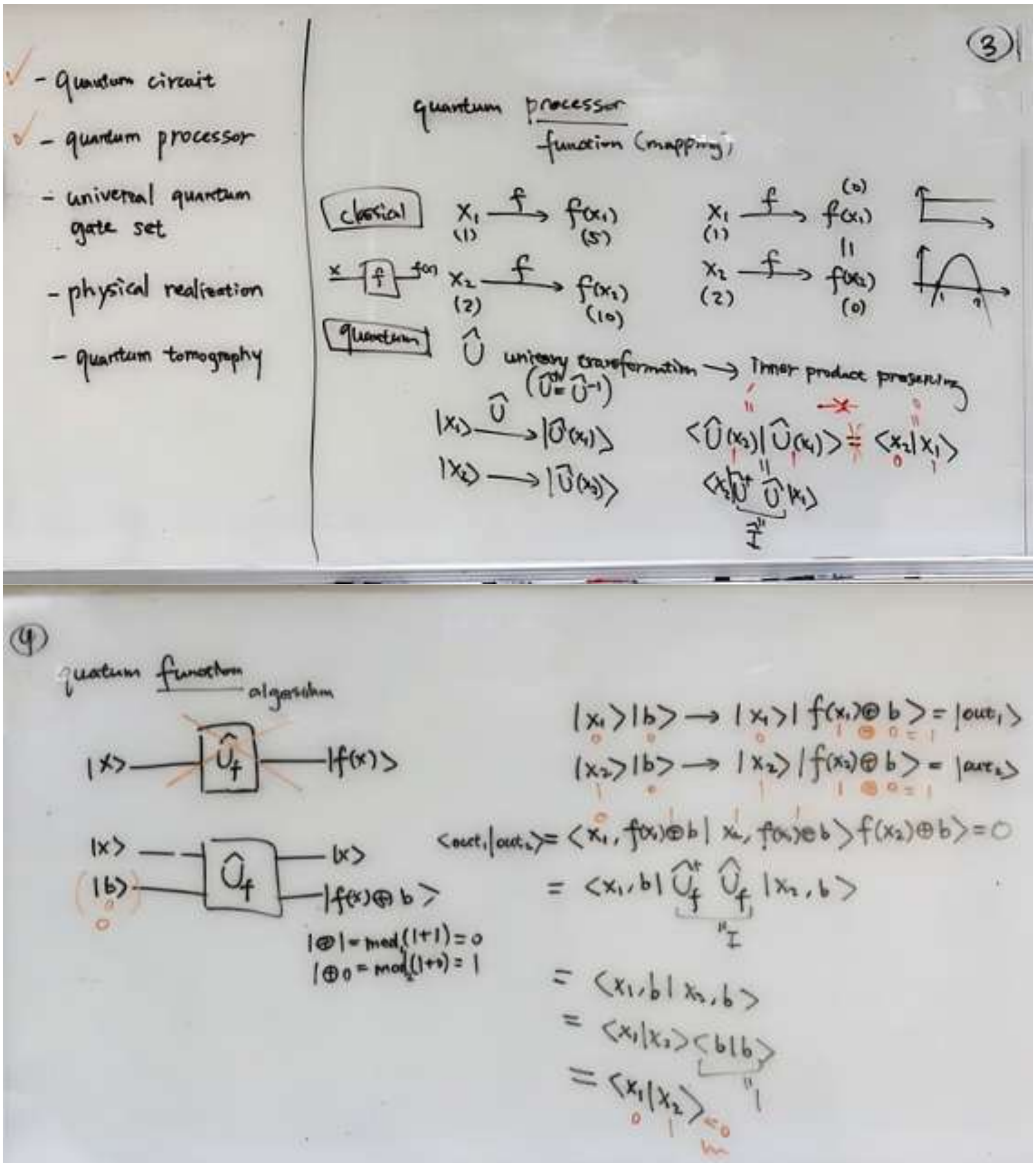
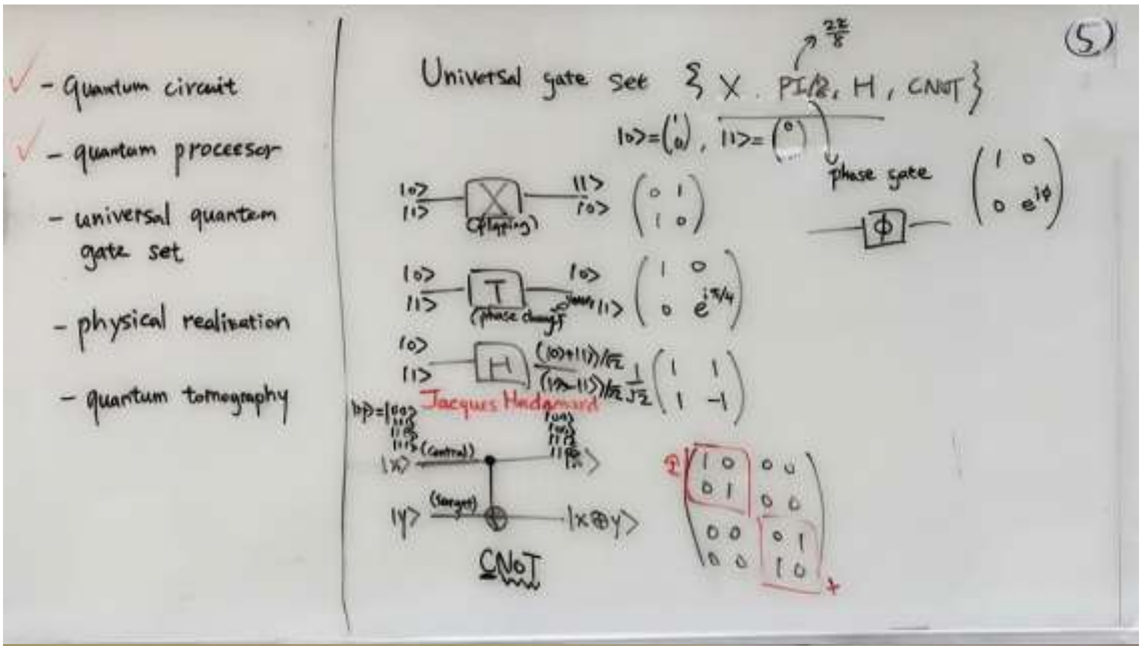
Lecture13 (spin) single-qubit gates and quantum state tomography






③
- quantum circuit
- quantum processor
- universal quantum gate set
- physical realization
- quantum tomography
quantum processor
function (mapping)
classical
quantum
unitary transformation → Inner product preserving
④
quantum function algorithm
|x⟩|b⟩ → |x⟩|f(x)⊕b⟩ = |out⟩
1⊕1 = mod(1+1) = 0
1⊕0 = mod(1+0) = 1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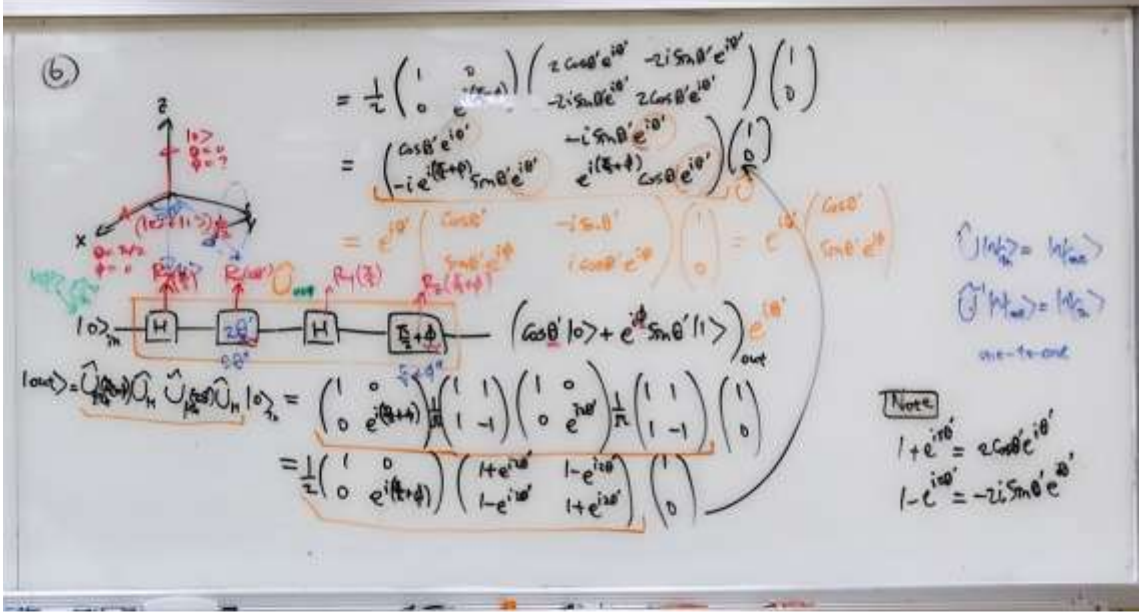
- Quantum circuit
- quantum processor
- universal quantum gate set
- physical realization
- quantum tomography
Universal gate set {X, PI/8, H, CNOT}
phase gate
(flipping)
(phase change)
Jacques Hadamard
(control)
(target)
CNOT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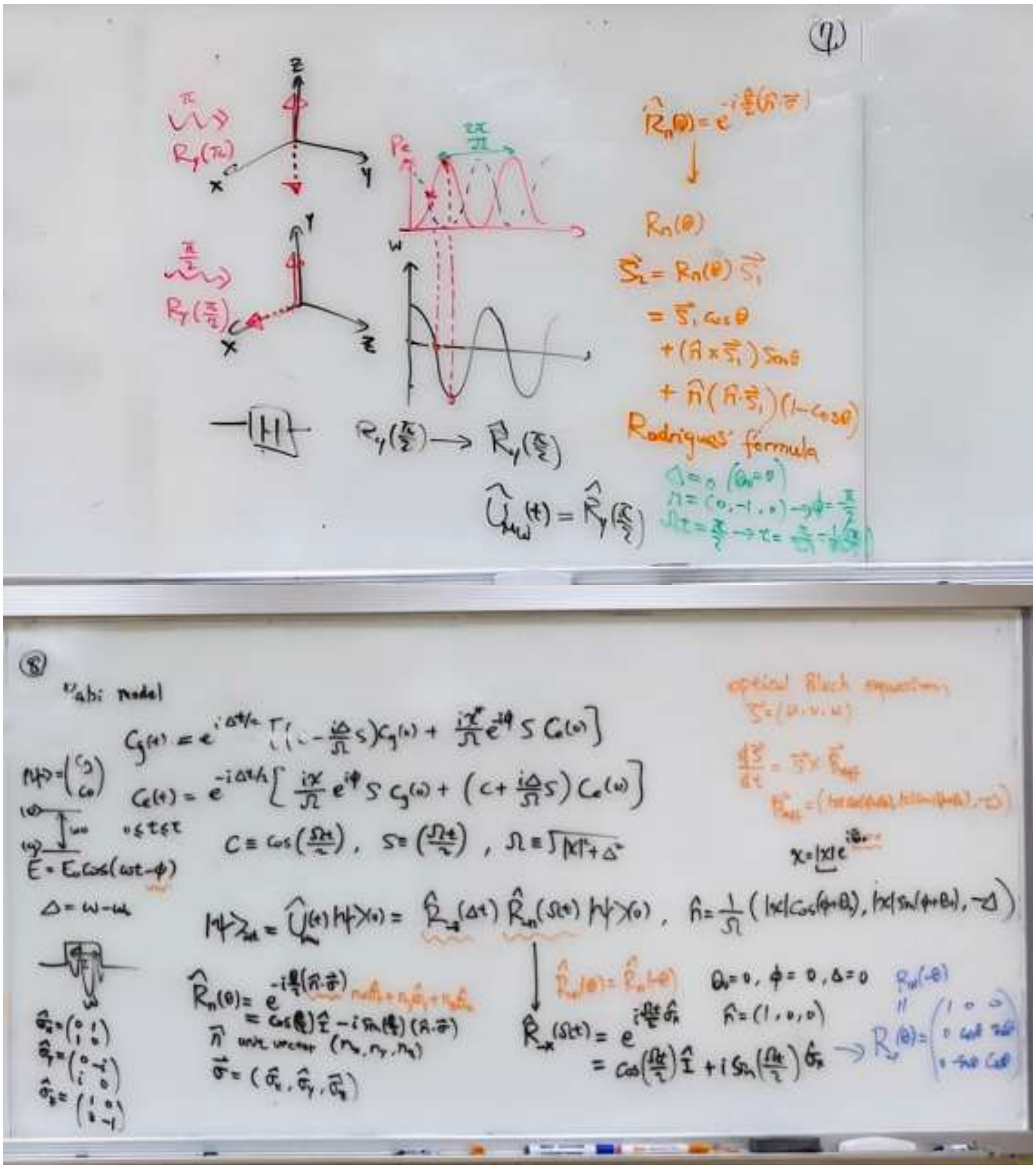
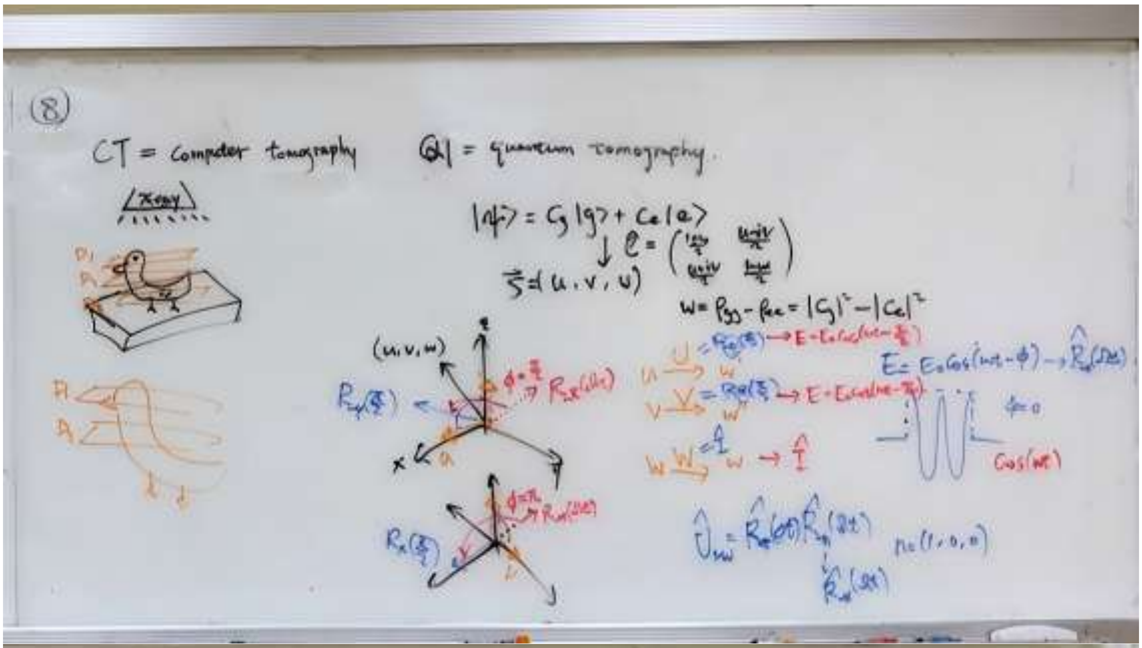
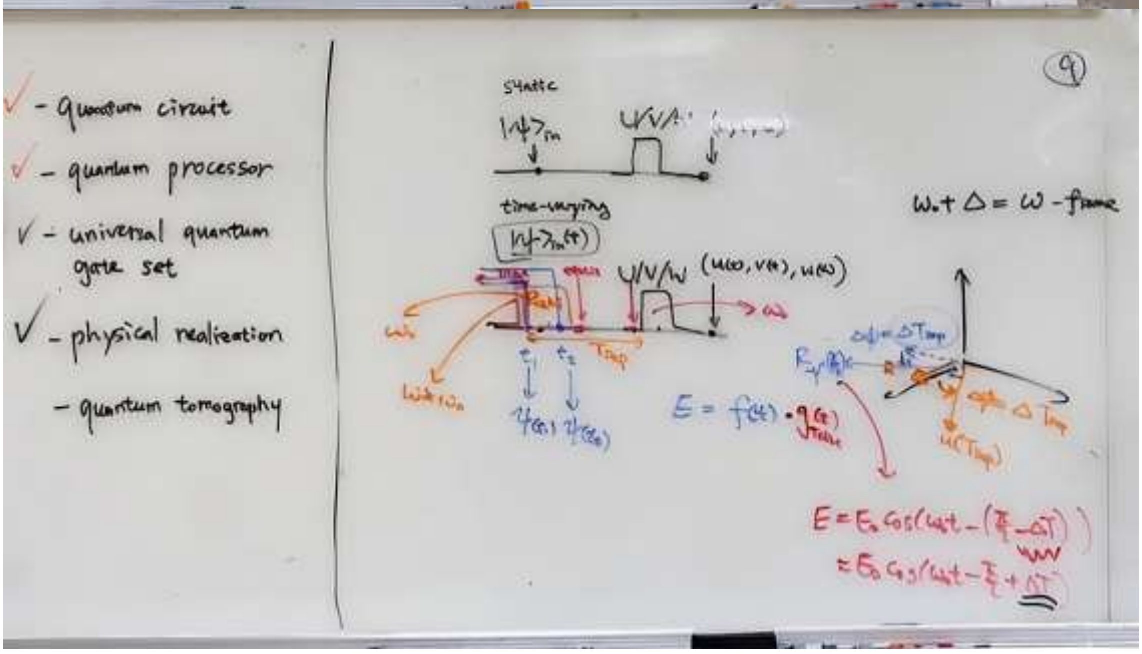
Note
one-to-one


⑦
$R_y(\pi)$
$R_y(\frac{\pi}{2})$
$\hat{R}_n(\theta) = e^{-i\frac{\theta}{2}(\hat{n}\cdot\vec{\sigma})}$
$R_n(\theta)$
$\vec{S}_2 = R_n(\theta)\vec{S}_1$
$= \vec{S}_1 \cos\theta$
$+ (\hat{n}\times\vec{S}_1)\sin\theta$
$+ \hat{n}(\hat{n}\cdot\vec{S}_1)(1-\cos\theta)$
Rodrigues' formula
$R_y(\frac{\pi}{2}) \longrightarrow \hat{R}_y(\frac{\pi}{2})$
$\hat{U}_{MW}(t) = \hat{R}_y(\frac{\pi}{2})$
⑧
Rabi model
$C_g(t) = e^{i\Delta t/2}\left[(c - \frac{i\Delta}{\Omega}s)C_g(0) + \frac{i\chi^*}{\Omega}e^{-i\phi}s\,C_e(0)\right]$
$C_e(t) = e^{-i\Delta t/2}\left[\frac{i\chi}{\Omega}e^{i\phi}s\,C_g(0) + (c + \frac{i\Delta}{\Omega}s)C_e(0)\right]$
$C \equiv \cos(\frac{\Omega t}{2})$, $S \equiv (\frac{\Omega t}{2})$, $\Omega \equiv \sqrt{|\chi|^2+\Delta^2}$
$E = E_0\cos(\omega t - \phi)$
$\Delta = \omega - \omega_0$
optical Bloch equation
$\chi = |\chi| e^{i\theta_0}$
$|\psi\rangle_{\text{int}} = \hat{U}(t)|\psi\rangle(0) = \hat{R}_z(\Delta t)\hat{R}_n(\Omega t)|\psi\rangle(0)$, $\hat{n} = \frac{1}{\Omega}(|\chi|\cos(\phi+\theta_0), |\chi|\sin(\phi+\theta_0), -\Delta)$
$\hat{R}_n(\theta) = e^{-i\frac{\theta}{2}(\hat{n}\cdot\vec{\sigma})}$
$= \cos(\frac{\theta}{2})\hat{I} - i\sin(\frac{\theta}{2})(\hat{n}\cdot\vec{\sigma})$
$\hat{n}$ unit vector (n_x, n_y, n_z)
$\vec{\sigma} = (\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z)$
$\theta_0 = 0,\ \phi = 0,\ \Delta = 0$
$\hat{n} = (1, 0, 0)$
$= \cos(\frac{\Omega t}{2})\hat{I} + i\sin(\frac{\Omega t}{2})\hat{\sigma}_x$


⑧
CT = computer tomography
QT = quantum tomography
x-ray
|ψ⟩ = Cg|g⟩ + Ce|e⟩
w = ρgg − ρee = |Cg|² − |Ce|²
(u, v, w)
E = E₀Cos(ωt − φ)
φ = 0
Cos(ωt)


- quantum circuit
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⑨
static
time-varying
U/V/W
(u(t), v(t), w(t))
ω₀ + Δ = ω - frame
E = f(t) · g(t)
E = E₀ Cos(ωt − (π/2 − ΔT))
= E₀ Cos(ωt − π/2 + ΔT)

✓ - Quantum circuit

✓ - quantum processor

✓ - universal quantum gate set

✓ - physical realization

✓ - quantum tomography