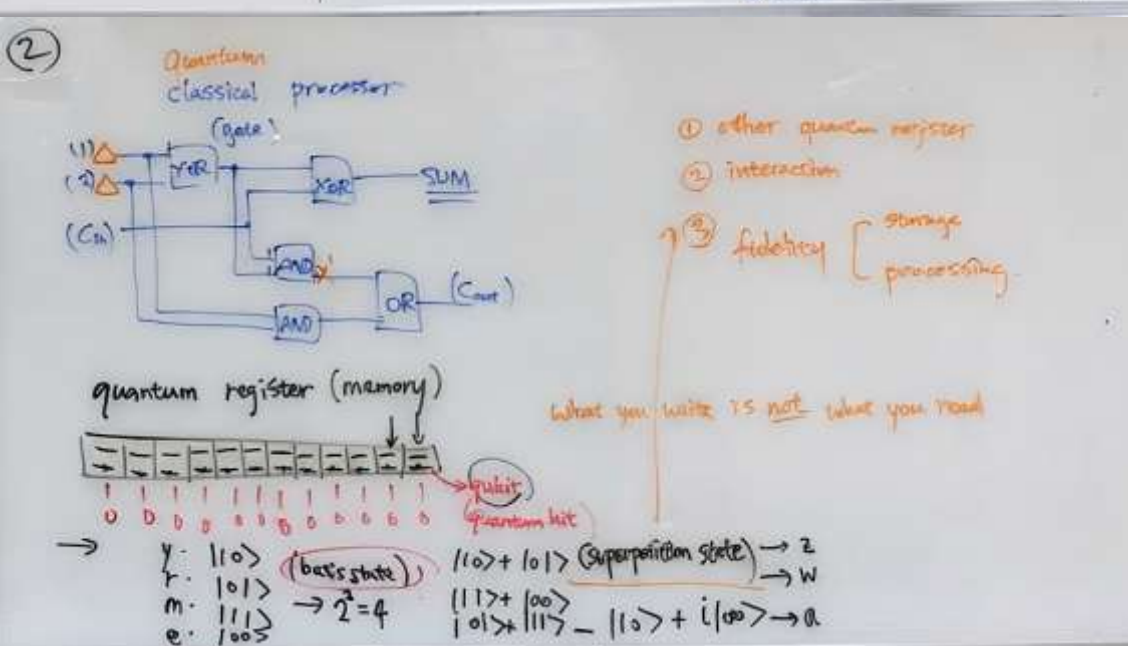
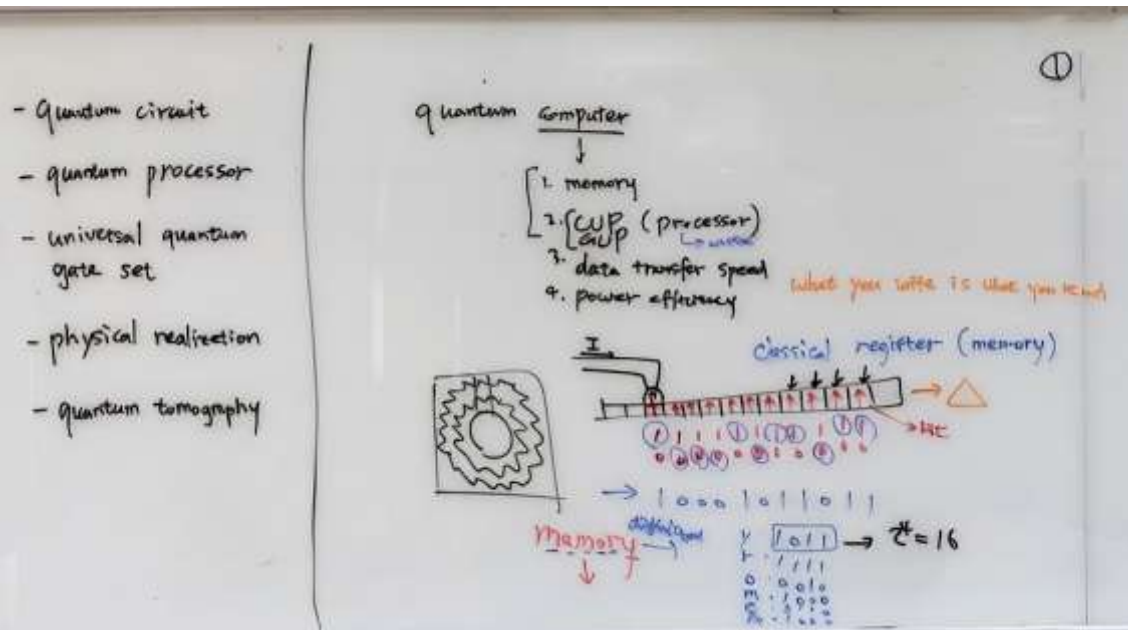




Lecture13 (spin) single-qubit gates and quantum state tomography





(3)

- ✓ - quantum circuit
- ✓ - quantum processor
- universal quantum gate set
- physical realization
- quantum tomography

quantum processor
function (mapping)

classical

$x_1 \xrightarrow{f} f(x_1)$
 $(1) \quad (5)$

$x_2 \xrightarrow{f} f(x_2)$
 $(2) \quad (10)$

$x_1 \xrightarrow{f} f(x_1)$
 $(1) \quad (5)$

$x_2 \xrightarrow{f} f(x_2)$
 $(2) \quad (10)$

quantum

$\hat{U}$ unitary transformation $\rightarrow$ Inner product preserving
 $(\hat{U}^\dagger = \hat{U}^{-1})$

$|x_1\rangle \xrightarrow{\hat{U}} |\hat{U}(x_1)\rangle$
 $|x_2\rangle \xrightarrow{\hat{U}} |\hat{U}(x_2)\rangle$

$\langle \hat{U}(x_2) | \hat{U}(x_1) \rangle = \langle x_2 | x_1 \rangle$
 $\langle \hat{U}(x_2) | \hat{U}(x_1) \rangle = \langle x_2 | x_1 \rangle$

(4) quantum function algorithm

$|x\rangle \xrightarrow{\hat{U}_f} |f(x)\rangle$

$|x_1\rangle |b\rangle \rightarrow |x_1\rangle |f(x_1) \oplus b\rangle = |out_1\rangle$
 $|x_2\rangle |b\rangle \rightarrow |x_2\rangle |f(x_2) \oplus b\rangle = |out_2\rangle$

$|x\rangle$
 $|b\rangle$
 $\hat{U}_f$
 $|x\rangle$
 $|f(x) \oplus b\rangle$

$\langle out_1 | out_2 \rangle = \langle x_1, f(x_1) \oplus b | x_2, f(x_2) \oplus b \rangle$
 $= \langle x_1, b | \hat{U}_f^\dagger \hat{U}_f | x_2, b \rangle$
 $= \langle x_1, b | x_2, b \rangle$
 $= \langle x_1 | x_2 \rangle \langle b | b \rangle$
 $= \langle x_1 | x_2 \rangle$

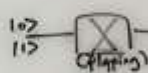
$| \oplus | = \text{mod}(1+1) = 0$
 $| \oplus 0 = \text{mod}(1+0) = 1$



- ✓ - Quantum circuit
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- quantum tomography

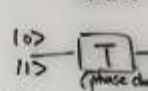
Universal gate set $\{X, \overset{\frac{2\pi}{8}}{\text{PI/8}}, H, \text{CNOT}\}$ (5)

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

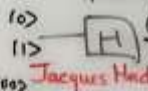


$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

phase gate $\begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}$

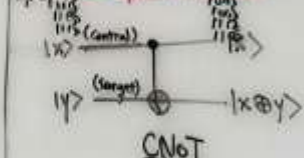


$$|0\rangle \rightarrow \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$$



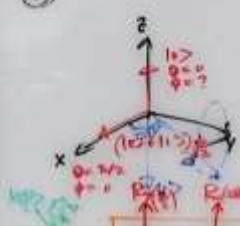
$$\frac{(10+11i)/\sqrt{2}}{(12-11i)/\sqrt{2}} = \frac{1}{j^2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$|b\rangle = |002\rangle$ Jacques Hadamard



$$P = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

(6)



$$= \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & e^{-j\pi/4} \end{pmatrix} \begin{pmatrix} 2 \cos \theta e^{j\theta} & -2j \sin \theta e^{j\theta} \\ -2j \sin \theta e^{j\theta} & 2 \cos \theta e^{j\theta} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} \cos \theta e^{i\phi} & -i \sin \theta e^{i\phi} \\ i \sin \theta e^{i\phi} & \cos \theta e^{i\phi} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$P(z) = e^{iz} \begin{pmatrix} \cos z & -i \sin z \\ \sin z e^{iz} & i \cos z e^{iz} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = e^{iz} \begin{pmatrix} \cos z \\ \sin z e^{iz} \end{pmatrix}$$


 $(\cos \theta |0\rangle + e^{i\phi} \sin \theta |1\rangle)$

$$|out\rangle = \hat{U}_{H_1} \hat{U}_{H_2} \hat{U}_{H_3} |0\rangle = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & e^{i(\frac{\pi}{2} + \phi)} \end{pmatrix} \begin{pmatrix} 1 + e^{i2\theta} & 1 - e^{i2\theta} \\ 1 - e^{i2\theta} & 1 + e^{i2\theta} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\hat{U} | \psi_k \rangle = | \psi_k \rangle$$

Chlorine

one-to-one

Note

Note

$$1 + e^{-j\omega} = 2 \cos \frac{\omega}{2}$$





⑦

$\hat{R}_n(\theta) = e^{-i\frac{\theta}{\hbar}(\hat{n} \cdot \vec{S})}$
 $\hat{R}_n(\theta)$
 $\hat{S}_z = \hat{R}_n(\theta) \hat{S}_z$
 $= \hat{S}_z \cos \theta$
 $+ (\hat{n} \times \hat{S}_z) \sin \theta$
 $+ \hat{n} (\hat{n} \cdot \hat{S}_z) (1 - \cos \theta)$
 Rodrigues' formula
 $\Delta = 0 \quad (\theta = 0)$
 $\hat{n} = (0, 1, 0) \rightarrow \phi = \frac{\pi}{2}$
 $\Delta = \frac{\pi}{2} \rightarrow \theta = \frac{\pi}{2}$

$\hat{U}_H(t) = \hat{R}_Y(\frac{\pi}{2})$

⑧

Abi Model

$G(t) = e^{i\Delta t/\hbar} \left[\left(1 - \frac{i\Delta}{\hbar} S\right) C_1(t) + \frac{i\Delta}{\hbar} e^{i\phi} S C_2(t) \right]$
 $C_2(t) = e^{-i\Delta t/\hbar} \left[\frac{i\Delta}{\hbar} e^{i\phi} S C_1(t) + \left(1 + \frac{i\Delta}{\hbar} S\right) C_2(t) \right]$
 $C \equiv \cos\left(\frac{\Omega t}{2}\right), S \equiv \left(\frac{\Omega t}{2}\right), \Omega \equiv \sqrt{|\vec{k}|^2 + \Delta^2}$
 $E = E_0 \cos(\omega t - \phi)$
 $\Delta = \omega - \omega_0$

optical Bloch equations
 $\dot{\vec{S}} = (\vec{\mu} \cdot \nabla) \vec{u}$
 $\frac{d\vec{S}}{dt} = \vec{S} \times \vec{B}_{eff}$
 $\vec{B}_{eff} = (\hbar \omega_0 / 2, \hbar \omega / 2, -\Delta)$
 $\chi = |\chi| e^{i\phi}$

$|\psi\rangle_{\text{out}} = \hat{U}(t) |\psi\rangle_{\text{in}} = \hat{R}_x(\Delta t) \hat{R}_n(\Omega t) |\psi\rangle_{\text{in}}, \hat{n} = \frac{1}{\Omega} (|\chi| \cos(\phi + \theta), |\chi| \sin(\phi + \theta), -\Delta)$
 $\hat{R}_n(\theta) = e^{-i\frac{\theta}{\hbar}(\hat{n} \cdot \vec{S})}$
 $= \cos\left(\frac{\theta}{2}\right) \hat{I} - i \sin\left(\frac{\theta}{2}\right) (\hat{n} \cdot \vec{S})$
 $\hat{n}$ unit vector (n_x, n_y, n_z)
 $\vec{S} = (\hat{S}_x, \hat{S}_y, \hat{S}_z)$
 $\hat{S}_x = \frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
 $\hat{S}_y = \frac{\hbar}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$
 $\hat{S}_z = \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$\hat{R}_x(\Omega t) = e^{-i\frac{\Omega t}{\hbar} \hat{S}_x}$
 $= \cos\left(\frac{\Omega t}{2}\right) \hat{I} + i \sin\left(\frac{\Omega t}{2}\right) \hat{S}_x \rightarrow \hat{R}_x(\theta) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix}$





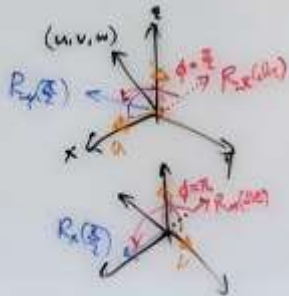
(8)

CT = Computer tomography

QT = Quantum tomography



$$|\psi\rangle = G|g\rangle + C_e|e\rangle$$
$$\rho = \begin{pmatrix} |G|^2 & GC_e^* \\ GC_e & |C_e|^2 \end{pmatrix}$$
$$\vec{S} = (u, v, w)$$



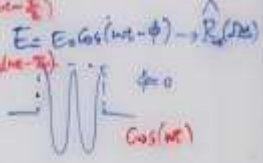
$$W = \rho_{gg} - \rho_{ee} = |G|^2 - |C_e|^2$$

$$u = \frac{G}{\sqrt{1+W}} \rightarrow E = E_0 \cos(\omega t - \frac{\pi}{2})$$

$$v = \frac{C_e}{\sqrt{1+W}} \rightarrow E = E_0 \cos(\omega t - \frac{\pi}{2})$$

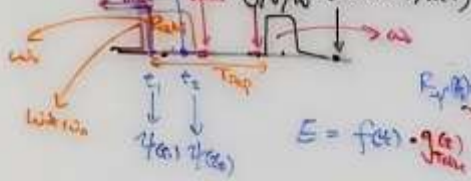
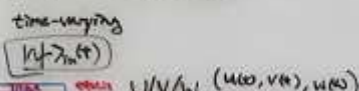
$$w = \frac{G^2 - C_e^2}{1+W} \rightarrow E = E_0 \cos(\omega t - \frac{\pi}{2})$$

$$\rho_{gg} = R_z(\theta) R_y(\phi) \rho_{gg} R_y(\phi) R_z(\theta)$$
$$\rho_{ee} = (1, 0, 0)$$



- ✓ - quantum circuit
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(9)



$$\omega + \Delta = \omega - \text{frame}$$

$$E = f(\omega) \cdot g(\omega)$$

$$E = E_0 \cos(\omega t - (\frac{\pi}{2} - \Delta t))$$
$$= E_0 \cos(\omega t - \frac{\pi}{2} + \Delta t)$$





- ✓ - Quantum circuit
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